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REDUCTION OF ELASTIC MODULI OF FIBER COMPOSITES BY FATIGUE CRACK ACCUMULATION

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The problem of reduction of elastic moduli of unidirectional fiber composites due to parallel cracks, either parallel or normal to fibers, is considered on the basis of variational techniques. Expressions for effective elastic moduli are given for the case of a small number of non-interacting cracks. It is shown that these are upper bounds on effective elastic moduli for the case of arbitrary distributions of parallel cracks. Lower bounds for Young's moduli for special parallel crack distributions are also constructed. In addition, the "self consistent scheme" approximation is also applied to the present problem.

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The problem treated is of significance for fatigue damage of fiber composites since such damage consists primarily of formation of such cracks. Experimental determination of moduli reduction due to cyclic stress (wearout) can thus provide a measure of the number of cracks formed.

1. INTRODUCTION

The present work is concerned with analytical determination of the effective elastic moduli of a unidirectional fiber composite which contains a distribution of parallel cracks, either in fiber direction or normal to the fibers (but not both kinds at the same time). The cracked material is viewed as a homogeneous orthotropic sheet which contains cracks, the orthotropic moduli being the effective elastic moduli of the fiber composite. The presence of cracks reduces these moduli to an extent to be determined.

This problem is related to quantification of fatigue damage in unidirectional fiber composites. During load cycling cracks appear and grow. The reduction of moduli after a number of cycles is often referred to as wearout. If a reliable prediction of moduli reduction in terms of crack distribution is available then experimental determination of such reduction, which is not difficult to accomplish, provides a measure of the extent of crack formation, thus of the damage and possibly of the residual strength.

The general problem of analytical determination of the elastic moduli of a cracked solid has received repeated attention, but not many exact results are available. Most of these are concerned with the case of a small number of non-interacting cracks in which situation the problem is easily solved [1-8]. The case of a periodic plane array of cracks arranged in a rectangular pattern has been treated by a combination of analytical and numerical methods in [9].

In addition, a general approximate method known as the "self consistent scheme"(SCS) has been applied to the case of randomly distributed elliptical cracks in an isotropic medium [10] as well as to the case of an oriented pattern of penny shaped cracks [11]. The basic assumption underlying the SCS is that any crack "sees" the effective homogeneous medium and thus the energy change due to the presence of any crack is computed as if this crack were placed in an infinite homogeneous medium whose elastic moduli are the effective moduli of the material with many cracks. This assumption is of questionable validity since any crack "sees" matrix and neighboring cracks. It is only on a sufficiently large scale of magnitude that the effective property concept becomes useful, thus for a region containing many cracks, not for a crack neighborhood.

As will be seen further on, computation of effective elastic moduli of an elastic body containing a distribution of cracks requires the determination of the energy change due to a crack in the presence of other interacting cracks. A general solution of this problem does not seem possible. It can be carried out analytically, for non-interacting cracks, or numerically, for a specific periodic crack geometry. Consequently, the approach adopted here is to construct bounds on the effective moduli by use of variational principles.

2. DIRECT APPROACHES

Consider a unidirectional reinforced layer specimen which contains a distribution of parallel cracks, Fig. 1. It is assumed that the layer is statistically homogeneous which implies

that any sufficiently large portion of it has the same effective elastic properties as the entire specimen.

The elastic moduli of the uncracked material are

E_A - Young's modulus in fiber direction

ν_A - Associated Poisson's ratio

E_T - Young's modulus transverse to fibers

G_A - Shear modulus

If the specimen is subjected to average plane stress its effective stress strain relations are

$$\begin{aligned} \bar{\varepsilon}_{11} &= \frac{\bar{\sigma}_{11}}{E_1^*} - \frac{\nu_{12}^*}{E_1^*}\bar{\sigma}_{22} \\ \bar{\varepsilon}_{22} &= -\frac{\nu_{12}^*}{E_1^*}\bar{\sigma}_{11} + \frac{\bar{\sigma}_{22}}{E_2^*} \\ \bar{\varepsilon}_{12} &= \frac{\bar{\sigma}_{12}}{2G_{12}^*} \end{aligned} \tag{2.1}$$

where overbars denote specimen averages and asteriks denote effective elastic moduli of cracked material.

A little reflection will show that

$$\begin{aligned} E^* &= E_A \\ \nu_{12}^* &= \nu_A \end{aligned} \tag{2.2}$$

since a homogeneous uniaxial stress field in x_1 direction and its associated strains are not affected by cracks in x_1 direction. Similarly, if the cracks were all parallel to the x_2 direction E_2^* and ν_{21}^* would be equal to E_T and ν_{TA}, respectively.

By the definition (2.1) it follows that in order to compute an effective elastic modulus it is necessary to compute the average strain due to an applied traction in the presence of cracks. In the case of a general distribution of cracks, this may be considered an intractable problem.

An alternative and equivalent definition is in terms of stored elastic energy. For applied stresses $\bar{\sigma}_{22}$, $\bar{\sigma}_{12}$, each separately, we have

$$U^{\sigma} = \frac{1}{2}\frac{\bar{\sigma}^2_{22}}{E_2^*} = \frac{1}{2}\frac{\bar{\sigma}^2_{22}}{E_T} + \Delta U^{\sigma} \tag{2.3}$$

$$\Delta U^{\sigma} = \sum^{n} \Delta U^{\sigma}_{n}$$

$$U^{\tau} = \frac{1}{2}\frac{\bar{\sigma}^2_{12}}{G_{12}^*} = \frac{1}{2}\frac{\bar{\sigma}^2_{12}}{G_A} + \Delta U^{\tau} \tag{2.4}$$

$$\Delta U^{\tau} = \sum^{n} \Delta U_n^{\tau}$$

where U^{σ} denotes stress energy per unit area of specimen and ΔU^{σ}_{n}, ΔU^{τ}_{n} are energy changes due to any one crack, in the presence of the others, in a tension or a shear field, respectively.

When the number of cracks is small and it can be assumed that their mutual interaction is negligible, the energy change ΔU^{σ}_{n} can be determined as if any crack were isolated in an infinite orthotropic sheet under the pertinent state of stress. In that case we have, [12].

$$\Delta U_n^{\sigma} = \pi a_n^2 \, \bar{\sigma}_{22}^2 \frac{1}{\sqrt{2E_T}} \left[\frac{1}{\sqrt{E_A E_T}} + \frac{1}{2G_A} - \frac{\nu_A}{E_A} \right]^{1/2} \tag{2.5}$$

$$\Delta U_n^{\tau} = \pi a_n^2 \bar{\sigma}_{12}^2 \frac{1}{\sqrt{2E_A}} \left[\frac{1}{\sqrt{E_A E_T}} + \frac{1}{2G_A} - \frac{\nu_A}{E_A} \right]^{1/2}$$

Consequently the small concentration results assume the forms:

$$\frac{E_2^*}{E_T} = \frac{1}{1+\pi \sqrt{2E_T} \left[\frac{1}{\sqrt{E_A E_T}} + \frac{1}{2G_A} - \frac{\nu_A}{E_A} \right]^{1/2} \alpha}$$

$$\frac{G_{12}^*}{G_A} = \frac{1}{1+\pi \frac{\sqrt{2G_A}}{\sqrt{E_A}} \left[\frac{1}{\sqrt{E_A E_T}} + \frac{1}{2G_A} - \frac{\nu_A}{E_A} \right]^{1/2} \alpha} \tag{2.6}$$

where

$$\alpha = \frac{1}{S} \sum_n a_n^2 \tag{2.7}$$

and S is the surface of the specimen. Thus α is a measure of the crack density per unit area.

According to the SCS approximation the energy change due to a crack is computed <u>as if</u> the crack were imbedded in the effective material. Therefore in this case we have

$$\Delta U_n^{\sigma} \approx \pi a_n^2 \bar{\sigma}_{22}^2 \frac{1}{\sqrt{2E_2^*}} \left[\frac{1}{\sqrt{E_1^* E_2^*}} + \frac{1}{2G_{12}^*} - \frac{\nu_{12}^*}{E_1^*} \right]^{1/2}$$

$$\Delta U_n^{\tau} \approx \pi a_n^2 \bar{\sigma}_{12}^2 \frac{1}{\sqrt{2E_1^*}} \left[\frac{1}{\sqrt{E_1^* E_2^*}} + \frac{1}{2G_{12}^*} - \frac{\nu_{12}^*}{E_1^*} \right]^{1/2} \tag{2.8}$$

which results in the following approximation relation for the effective moduli E_2^* and G_{12}^*, [13].

$$\frac{E_2^*}{E_T} = 1 - \pi\sqrt{2E_2^*}\left[\frac{1}{\sqrt{E_A E_2^*}} + \frac{1}{2G_{12}^*} - \frac{\nu_A}{E_A}\right]^{1/2}\alpha$$

$$\frac{G_{12}^*}{G_A} = 1 - \pi\frac{\sqrt{2}\,G_{12}^*}{\sqrt{E_A}}\left[\frac{1}{\sqrt{E_A E_2^*}} + \frac{1}{2G_{12}^*} - \frac{\nu_A}{E_A}\right]^{1/2}\alpha \tag{2.9}$$

3. VARIATIONAL APPROACHES

Bounds on the effective elastic moduli can be obtained by use of the elasticity extremum principles. The elastic energies (2.3-4) can be bounded by use of the principles of minimum potential and minimum complementary energy. For this purpose it is necessary to construct suitable admissible fields for displacements or for stresses.

Consider a rectangular cracked specimen under uniaxial stress transverse to the cracks, Fig. 2. The boundary conditions are

$$\begin{aligned} \sigma_{12}\,(\pm\ell_1,\, x_2) &= 0 \\ \sigma_{22}\,(x_1,\, \pm\ell_2) &= \sigma_0 \\ \sigma_{12}\,(x_1,\, \pm\ell_2) &= 0 \end{aligned} \tag{3.1}$$

On all crack surfaces

$$\sigma_{22} = \sigma_{12} = 0 \tag{3.2}$$

An admissible desplacement $\tilde{u}_i(x_1,x_2)$ field must be continuous everywhere and satisfy all displacement boundary conditions. In the present case the boundary conditions are (3.1-2), thus all four transactions, and therefore an admissible displacement must merely be continuous.

An admissible stress field $\tilde{\sigma}_{ij}(x_1,x_2)$ must satisfy equilibrium

everywhere and the traction boundary conditions (3.1-2) Therefore the construction of admissible stress fields is much more difficult than that of admissible displacement fields, in the present case.

The displacement field

$$\tilde{u}_i = u_i^{o} + \sum_{n=1}^{N} u_{in}' \tag{3.3}$$

where u_i^{o} are the displacements due to the applied stresses in the uncracked body and u_{in}' the perturbation displacement field of the n-th crack as if it were isolated in an infinite body, is an admissible displacement field for the given problem, since it is continuous everywhere. It is shown in Appendix A that this field leads to the result that the small concentration approximations (2.6) are upper bounds on the effective elastic moduli for any crack distribution. Thus

$$\frac{E_2^*}{E_T} \leq \frac{1}{1 + \pi\sqrt{2E_T}\left[\frac{1}{\sqrt{E_A E_T}} + \frac{1}{2G_A} - \frac{\nu_A}{E_A}\right]^{1/2}\alpha}$$

$$\frac{G_{12}^*}{G_A} \leq \frac{1}{1 + \pi\frac{\sqrt{2G_A}}{\sqrt{E_A}}\left[\frac{1}{\sqrt{E_A E_T}} + \frac{1}{2G_A} - \frac{\nu_A}{E_A}\right]^{1/2}\alpha} \tag{3.4}$$

In order to obtain lower bounds an admissible stress field has to be constructed. If we divide the specimen of Fig. 2 into smaller rectangles, each one containing one central crack,

the solution of the problem of one such finite cracked rectangle under uniaxial tension (Fig. 3) is an admissible stress field for the problem described in Fig. 2, since it satisfies equilibrium and traction boundary conditions on all the cracks.

The problem of Fig. 3 was solved numerically [14] and the stress intensity factor K was given as a function of the geometrical parameters a, b and c and the elastic properties of the orthotropic body. Using this result in the principle of minimum complementary energy as described in Appendix B, a lower bound on E_2^* is obtained.

$$\frac{E_2^*}{E_T} \geq \frac{1}{1 + \pi \sqrt{2E_T} \left[\frac{1}{\sqrt{E_A E_T}} + \frac{1}{2G_A} - \frac{\nu_A}{E_A} \right]^{1/2} f^2 \alpha} \tag{3.5}$$

where f implies f(a,b,c, material properties) and is given in [14].

It appears, that a corresponding solution of a finite centrally cracked rectangle under pure shear is not available; thus no lower bound on G_{12}^* can be given here.

4. DISCUSSION

It has been shown that the upper bounds for the effective elastic moduli of unidirectional composites can be determined by use of the variational theorems of the theory of elasticity. These bounds are general and easy to calculate. The lower bounds are more problematic and can be contructed only for special cases.

All of the results obtained for elastic moduli of a cracked orthotropic layer are illustrated by application to a typical

glass fiber-polyester matrix unidirectional composite.

The elastic properties of the uncracked material are:

E_A = 26.4 GPa; E_T = 7.17 GPa; G_A = 5.12 GPa; ν_A = .267

Using these constants in Eq. (2-9), (3-4), and (3-5) the various results for E_2^*/E_A and G_{12}^*/G_A as a function of α have been obtained (Fig. 4 and Fig. 5). For E_2^*/E_T upper and lower bounds have been constructed. However only an upper bound is available for G_{12}^*/G_A.

In the case E_2^* upper and lower bounds are reasonably close; the lower bound being higher than the SCS approximation. The lower bound and G_{12}^* is very close to the SCS approximation.

The present results can be incorporated into analysis of laminates with cracked layers by use of the effective moduli results for any one cracked layer.

Measurement of reduction of the effective elastic moduli (wearout) during cycling can, in conjunction with present results, serve to estimate the crack damage in layers by evaluation of the α parameters.

REFERENCES

1. BRISTOW, J. R., British Journal of Applied Physics, Vol. 11, 1960, pp. 81-85.

2. WALSH, J. B., Journal of Geophysical Research, Vol. 70, 1965, pp. 381-389.

3. WALSH, J. B., Journal of Geophysical Research, Vol. 70, 1965, pp. 399-411.

4. WALSH, J. B., Journal of Geophysical Research, Vol. 70, 1965, pp. 5249-5257.

5. SALGANIC, I. L., IZV. AN SSSR, MIT (Mechanics of Solids), Vol. 4, 1973, pp. 136-143.

6. GARBIN, D.". and KNOPOFF, L., Quarterly of Applied Mathematics, Vol. 30, 1973, pp. 453-464.

7. GARBIN, D. H. and KNOPOFF, L., Quarterly of Applied Mathematics, Vol. 33, 1975, pp. 296-300.

8. GARBIN, D. H. and KNOPOFF, L., Quarterly of Applied Mathematics, Vol. 33, 1975, pp. 301-303.

9. DELAMETER, W. R., HERMANN, G. and BARNETT, D. M., Journal of Applied Mechanics, Vol. 42, 1975, pp. 76-80; ERRATUM: Journal of Applied Mechanics, Vol. 44, 1977, p. 190.

10. BUDIANSKY, B. and O'CONNELL, R. J., Int. J. Solids Structures, Vol. 12, 1976, pp. 81-97.

11. HOENIG, A., Int. J. Solids Structures, 15, 137-154, (1979).

12. SIH, G. C., PARIS, P. C. and IRWIN, G. R., Int. J. of Fracture Mechanics, Vol. 1, 1965, pp. 189-203,

13. HASHIN, Z. and DERBY, E., Materials Science Corp., Blue Bell, PA., 19422 TFR6301/1010, Nov. 1979.

14. BOWIE, O. L. and FREESE, C. E., Int. J. of Fracture Mechanics, Vol. 8, 1972, pp. 49-58.

15. SNEDDON, I. N. and LOWENGRUB, M., Crack Problems in the Classical Theory of Elasticity, John Wiley & Sons, Inc., 1969.

APPENDIX A

UPPER BOUNDS FOR EFFECTIVE MODULI

The displacement solution of one crack in an infinite body under tension is well known [15] and can be expressed as

$$u_i^{\infty} = u_i^{o} + u_i' \tag{A-1}$$

where u_i^{o} is the displacement associated with the applied stress in the uncracked body and u_i' is the perturbation field due to the crack. The displacement perturbation u_i goes to zero at infinity and is continous everywhere in the body.
It follows that

$$\tilde{u}_i = u_i^{o} + \sum_{n=1}^{N} (u_i')_n \tag{A-2}$$

is an admissible displacement field for the problem of the body containing N cracks since it is evidently continuous everywhere and boundary conditions on the cracks need not be satisfied since these are traction free, thus a traction prescribed boundary.

The strain and stress fields, associated with $\tilde{u}_i$ are

$$\tilde{\varepsilon}_{ij} = \frac{1}{2} (\tilde{u}_{ij} + \tilde{u}_{j\ i}) = \varepsilon_{ij}^{o} + \sum_{n=1}^{N} (\varepsilon_{ij}')_n \tag{A-3}$$

and

$$\tilde{\sigma}_{ij} = C_{ijkl}\, \tilde{\varepsilon}_{ij} = \sigma_{ij}^{o} + \sum_{n=1}^{N} (\sigma_{ij}')_n \tag{A-4}$$

The potential energy functional can be expressed as

$$\tilde{U}_p = \frac{1}{2} \int_V \tilde{\sigma}_{ij}\, \tilde{\varepsilon}_{ij}\, d\nu - \int_{S_T} \sigma_{ij}^{o}\, n_j\, \tilde{u}_i\, ds \tag{A-5}$$

Substituting (A-2), (A-3) and (A-4) into (A-5) it can be shown

that

$$\tilde{U}_p = - U_o - \sum_n \Delta U_n^{\infty} \tag{A-6}$$

where U_o is the strain energy of the uncracked body and ΔU_n^{∞} is the potential energy release due to one crack of length a_n in an infinite body. The substitution of Eq. (A-6) into the inequality

$$\tilde{U}_p \geq U_p \tag{A-7}$$

which is essentially the principle of minimum potential energy, leads to the conclusion that the results for small concentration of cracks are upper bounds on the effective moduli for any crack distribution.

APPENDIX B

As explained in the text the solution to the problem of Fig. 3 is an admissible stress field, for the problem of Fig. 2. From [14] the stress intensity factor is known as a function of the length of the crack, the dimensions of the rectangle and the elastic moduli of the uncracked body. For a rectangle of dimensions b_n, c_n containing a crack of length a_n this result is

$$K_n = \sigma_o \sqrt{\pi a_n}\; f_n\,(a_n, b_n, c_n, \text{ elastic moduli}) \tag{B-1}$$

The increase in stress energy functional (which is a special case of the complementary energy functional) due to the cracks is thus [12]

$$\Delta U^\sigma = \sum_n \pi\, a_n K_n^2 \frac{1}{\sqrt{2E_T}} \left[\frac{1}{\sqrt{E_A E_T}} + \frac{1}{2G_A} - \frac{\nu_A}{E_A} \right]^{1/2} \tag{B-2}$$

The stress energy functional can be written as

$$\tilde{U}^\sigma = U_o^\sigma + \Delta U^\sigma \tag{B-3}$$

where U_o^σ is the stress energy of the cracked body. Introducing (B-3) into the minimum stress energy (complementary energy) principle

$$U_\sigma \leq \tilde{U}_\sigma \tag{B-4}$$

and using (2.3) and (2.5) the bound (3.5) follows.

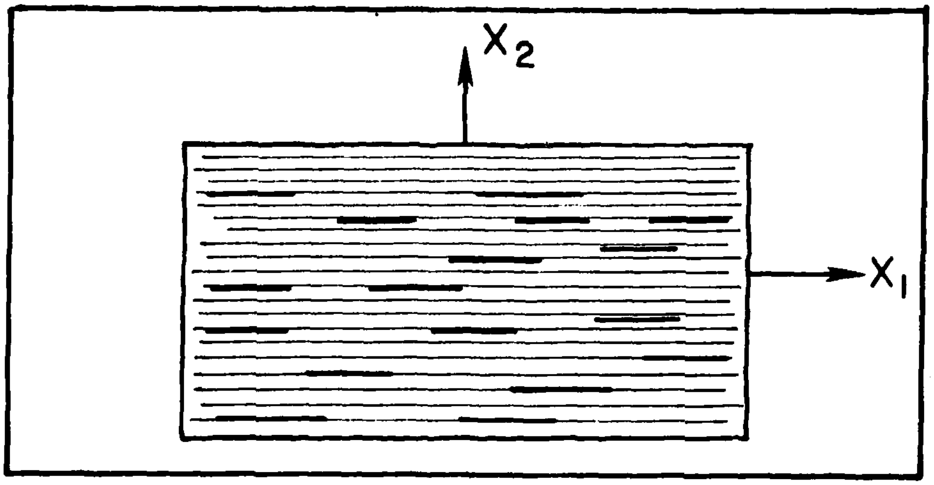

Fig. 1. Cracked Unidirectional Composite

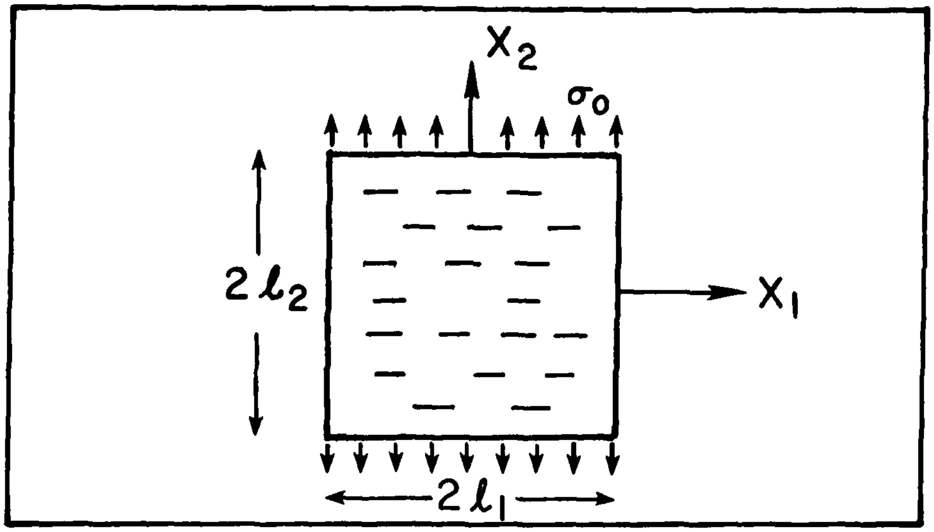

Fig. 2. Cracked Specimen under Transverse Stress

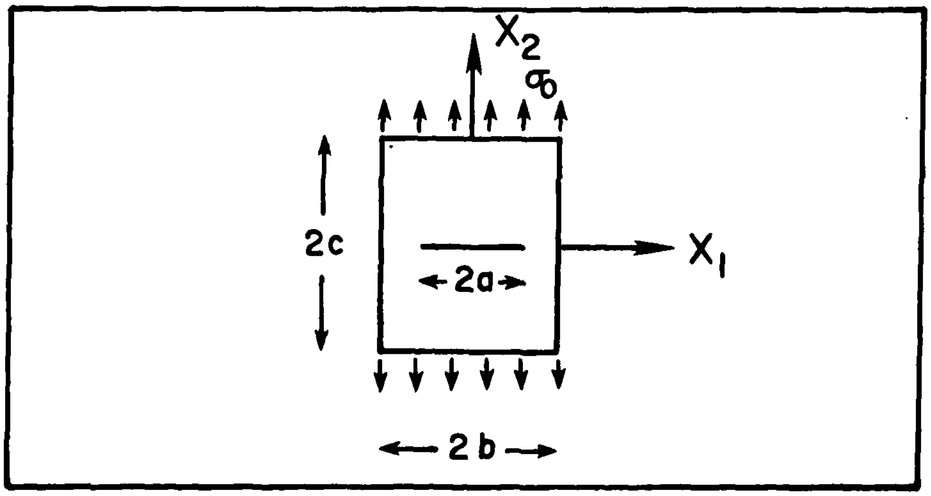

Fig. 3. Finite Rectangle with a Central Crack

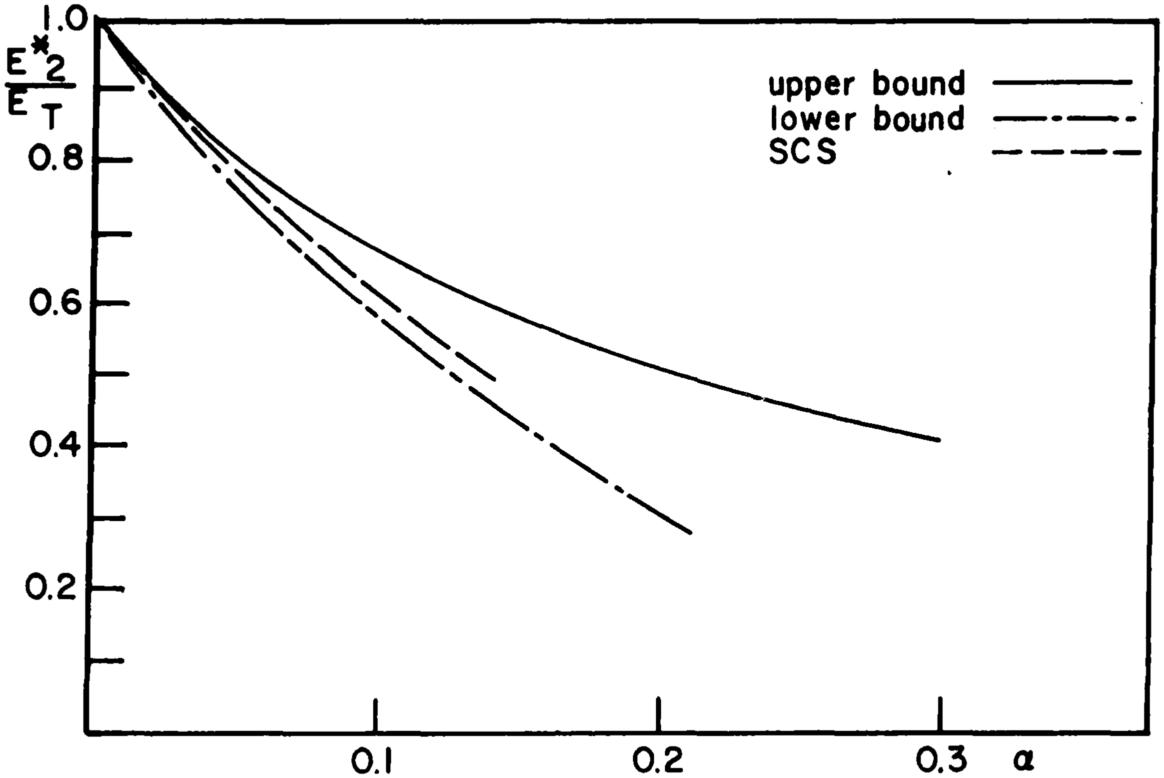

Fig. 4 Results for E^*_2 and ν^*_{21}. Glass-Polyester.

$$\nu^*_{21} = \nu_A \cdot \frac{E^*_2}{E_A}$$

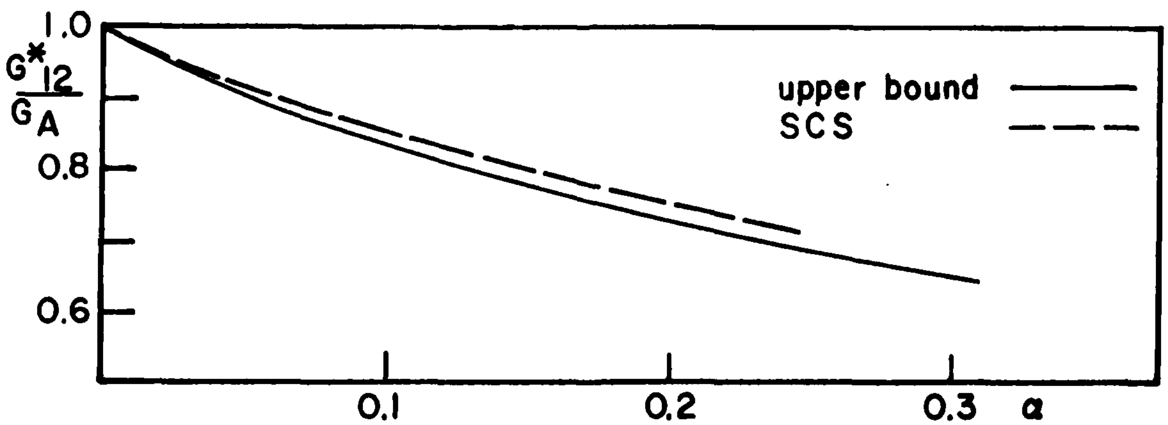

Fig. 5 Results for G^*_{12}. Glass-Polyester.